

# Mathematical Induction

1. Prove the following by mathematical induction:

a)  $1 + 3 + 5 + 7 + \dots + 2(n - 1) = n^2$  for all  $n \geq 1$

b)  $5 + 10 + 15 + \dots + 5n = \frac{5}{2}n(n + 1)$

c)  $1 + 9 + 25 + \dots + (2n - 1)^2 = \frac{1}{3}n(4n^2 - 1)$

d)  $1 + a + a^2 + \dots + a^{n-1} = \frac{1 - r^n}{1 - r}$

e)  $1 - 4 + 3 - \dots + (-1)^n n^2 = \frac{1}{2}(-1)^{n-1}n(n + 1)$

2. Prove that  $9^n - 1$  is divisible by 8 for all  $n \geq 1$  where  $n$  is an integer.

3. Prove by induction that  $2^n \geq n$  for all  $n \geq 1$

4. Prove that  $9^{n+1} - 8n - 9$  is divisible by 64 for all  $n \geq 1$

## Harder Questions

5. Prove by induction that:  $\cos(x + n\pi) = (-1)^n \cos(x)$

6. 
$$\sum_{r=1}^n (2r - 1)^3 = n^2(2n^2 - 1)$$